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Reverse Super Edge Magic Strength of Bistars

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Abstract

A graph G is said to be reverse super edge-magic if there exists a bijection f: $V \cup E \rightarrow \{1, 2, ..., p+q\}$ such that $f(uv) - [f(u) + f(v)]$ is a constant, for all $uv \in E$ and $f(V) = \{1, 2, ..., p\}$. Such a bijection is called a reverse super edge-magic labeling and the minimum of all constants is called a reverse super edge-magic strength of the graph G, where the minimum is taken over all reverse super edge-magic labelings of G. In this paper, the reverse super edge-magic labelings and reverse super edge-magic strength of the bistar $B_{m,\ n}$ and the tree $< K_{1,\ m} : K_{1,\ n} >$ obtained from Bistar $B_{m,\ n}$ are obtained.

Introduction

The graphs considered here are only finite, simple, undirected graphs. The basic notations and terminologies are used as in [3]. In particular, q is the number of edges in G.

Kotzig and Rosa [4] introduced the concept of magic valuation. Ringel and Llado′ [8] called this type of valuation as edge-magic labeling. Further Enomoto et al [2] restricted the notion of edge-magic labeling of a graph to obtain the definition of super edge-magic labeling. An edge- magic labeling of a graph G is called a super edge-magic labeling of G if $f(V)=\{1, 2, \ldots, p\}$ and $f(E) = \{p+1, p+2, \ldots, p+q\}$. A graph G is super edge-magic if it has super edge-magic labeling. The concept of a super edge–magic strength of a super edge-magic graph is introduced by Avadayappan et al [1]. They established the super edge-magic strength of the path P_{2n} of odd length, P_{2n+1} of even length, the star $K_{1,\ m}$, the bistar $B_{n,\ n}$, the tree $< K_{1,\ n} : 2 >$, the graph $(2n+1)P_2$ and the graph $P_n^{(2)}$.

In this paper, the new concepts of reverse super edge- magic labeling as well as reverse super edge-magic strength of a graph are introduced. The reverse super edge-magic labeling is a bijection f: $V \cup E \to \{1, 2, ..., p+q\}$ such that $f(V) = \{1, 2,..., p\}$, $f(E) = \{p+1, p+2, ..., p + q\}$ and $f(uv) - [f(u) + f(v)]$ is constant, for every edge uv of E. A graph G is said to be reverse super edge-magic if there exists such a bijection f of G (Fig.1.1).

Again for any reverse super edge-magic labeling f of G, there is a constant $c^{-1}(f)$, called reverse super edge magic strength, such that $f(uv) - [f(u) + f(v)] = c^{-1}(f)$, for any edge $uv \in E$. The reverse super edge-magic strength is denoted by rsems (G) and is defined as the minimum of all $c^{-1}(f)$, where the minimum is taken over all reverse super edge-magic labeling f of G.

Therefore rsems (G) = min $\{c^{-1}(f)$: f is a reverse super edge-magic labeling of G$\}$.

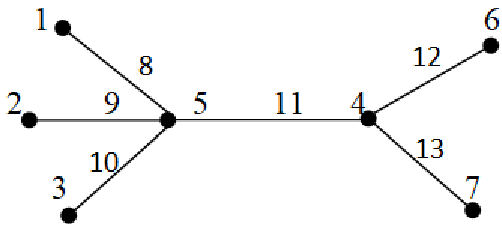

Fig 1.1: $B_{3,2}$ – bistar

Here the reverse super edge-magic labeling as well as reverse super edge-magic strength of some well known graphs such as the bistar $B_{m,n}$, the tree $< K_{1,m}: K_{1,n} >$ are established.

To proceed further, the following definations and results are useful.

***Definition* 1.1:** The graph $B_{m,n}$ is bi-star obtained from $K_{1,m}$ and $K_{1,n}$ by joining the vertices of maximum degree by an edge uv.

***Definition* 1.2:** The graph $< K_{1,m} : K_{1,n} >$ is a tree obtained from the bistar $B_{m,n}$ by subdividing the middle edge uv by a new vertex w.

***Note* 1.3:** Let f be any arbitrary reverse super edge-magic labeling of a graph G, with reverse super edge-magic constant $c^{-1}(f)$.

Then by adding all the constants obtained at each edge, we get

$$q\, c^{-1}(f) = \sum_{e \in E} f(e) - \sum_{v \in V} d(v) f(v)$$

2. Main Results

Theorem 2.1: rsems $(B_{m,n}) = n$, for $m \geq 1, n \geq 1$.

Proof: The tree (m, n)-bistar $B_{m,n} \cong G$ is defined as follows:

$V = \{ u_i, v_j : 0 \leq i \leq m, 0 \leq j \leq n \}$ and

$E = \{u_iu_0, u_0v_0, v_0v_j : 1 \leq i \leq m, 1 \leq j \leq n\}$

Consider the function f: $V \cup E \rightarrow \{1, 2, \ldots, (2m + 2n + 3)\}$ defined by

$$f(x) = \begin{cases} i, & \text{if } x = u_i,\ 1 \leq i \leq m \\ m+2, & \text{if } x = u_0 \\ m+1, & \text{if } x = v_0 \\ m+2+j, & \text{if } x = v_j,\ 1 \leq j \leq n. \end{cases}$$

and

$$f(e) = \begin{cases} m+n+2+i, & \text{if } e = u_iu_0,\ 1 \leq i \leq m \\ 2m+n+3, & \text{if } e = u_0v_0 \\ 2m+n+3+j, & \text{if } e = v_0v_j,\ 1 \leq j \leq n \end{cases}$$

Then clearly f is a reverse super edge- magic labeling with reverse super edge-magic constant $c^{-1}(f) \leq n$.

Now consider, by note1.3,

$$q\, c^{-1}(f) = \sum_{e \in E} f(e) - \sum_{v \in V} d(v)\, f(v)$$

$(m + n + 1)\, c^{-1}(f)$

$$= \{\sum_{i=1}^{m} (m + n + 2 + i) + (2m + n + 3) + \sum_{j=1}^{n} (2m + n + 3 + j)\} -$$

$$[\sum_{i=1}^{m} i + (m+1)(m+2) + (n + 1)(m + 1) + \sum_{j=1}^{n} ((m + 2) + j)]$$

$$= (m + n + 2) + (2m + n + 3) + (2m + n + 3)\, n - (m + 1)(m + 2) - (n + 1)(m + 1) - (m + 2)\, n$$

$$\geq n^2 + mn + n.$$

$$= n(m + n + 1)$$

$$\therefore c^{-1}(f) \geq n$$

Hence rsems $(B_{m,n}) = n$, for $m \geq 1$ and $n \geq 1$.

Corollary 2.1.a: rsems ($B_{n, n}$) = n, for $n \geq 1$.

Theorem 2.2: The tree $< K_{1, m} : K_{1, n} >$ is reverse super edge-magic and

rsems ($< K_{1, m} : K_{1, n} >$) = m + n, for $m \geq 1$ and $n \geq 1$.

Proof: The tree $G \cong < K_{1, m} : K_{1, n} >$ is defined on

$$V = \{u_i, u, w, v, v_j : 1 \leq i \leq m, 1 \leq j \leq n\}$$

and $$E = \{u_iu, uw, wv, vv_j : 1 \leq i \leq m, 1 \leq j \leq n\}$$

Now consider the function f: $V \cup E \rightarrow \{1, 2, \ldots, (2m + 2n + 5)\}$ defined by

$$f(x) = \begin{cases} 2+i, & if\ x = u_i \\ 1, & if\ x = u \\ m+3, & if\ x = w \\ 2, & if\ x = v \\ m+3+j, & if\ x = v_j \end{cases}$$

and $$f(e) = \begin{cases} m+n+3+i, & if\ e = u_iu \\ 2m+n+4, & if\ e = uw \\ 2m+n+5, & if\ e = wv \\ 2m+n+5+j, & if\ e = vv_j \end{cases}$$

where $1 \leq i \leq m$ and $1 \leq j \leq n$

Then clearly f is reverse super edge-magic labeling with reverse super edge-magic constant $c^{-1}(f) \leq m + n$.

Now to find the reverse super edge-magic strength of G,

consider $q\, c^{-1}(f) = \sum_{e \in E} f(e) - \sum_{v \in V} d(v)\, f(v)$

$$\therefore\ (m + n + 2)\, c^{-1}(f)$$

$$= \{\sum_{i=1}^{m} (m + n + 3 + i) + (2m + n + 4) + (2m + n + 5) + \sum_{j=1}^{n} (2m + n + 5 + j)\} -$$

$$[\sum_{i=1}^{m} (2 + i) + (m + 1)1 + 2(m + 3) + (n + 1)2 + \sum_{j=1}^{n} (m + 3 + j)]$$

$$= (m + n + 3)m + (4m + 2n + 9) + n(2m + n + 5) - 2m - m - 1 - 2(m+3) - 2(n + 1) - n(m + 3)$$

$$\geq m^2 + 2m + 2mn + 2n + n^2$$

$= (m + n)(m + n + 2)$

$\therefore c^{-1}(f) \geq m + n$

Hence rsems $(< K_{1,m} : K_{1,n} >) = m + n$, for $m \geq 1$ and $n \geq 1$.

Corollary 2.2.a: rsems $(< K_{1,n} : K_{1,n} >) = 2n$, for $n \geq 1$.

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